1) a) sample size N ✓

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

b) $$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i^2 - \bar{x}^2)$$ ✓

c) error in the mean

$$\sigma_\mu = \sqrt{\frac{1}{\sum_{i=1}^{N} 1/\sigma_i^2}}$$ but ~~when~~ if we assume $\sigma_i = \sigma$ ✓

$$\Rightarrow \quad \sigma_\mu = \sqrt{\frac{\sigma}{N}}$$

2) a) Gaussian distr mean $= \mu$ variance $= \sigma^2$ (std dev $= \sigma$) ✓

$$P_G(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\mu-x)^2}{2\sigma^2}}$$

μ and σ parameters of the distribution

b) Poissonian distr mean $= \mu$ variance $= \mu$ (std dev $= \sqrt{\mu}$) ✓

$$P_P(x;\mu) = \frac{\mu^x}{x!} e^{-\mu}$$

μ parameter of the distribution

c) Poisson and Gauss are both limit cases of the binomial distribution.

1/2 Poisson ~~is~~ when the chance of succes is very small ($p \ll 1$)$\frac{1}{2}$. and numb of cases is lar

1/2 ~~Gauss when~~ Poisson tends to Gaussian when the ~~rate~~ chance of success is bigger, than what?
and when the variance $\mu = \sigma^2$

Poisson tends to Gaussian when mean $\to \infty$.

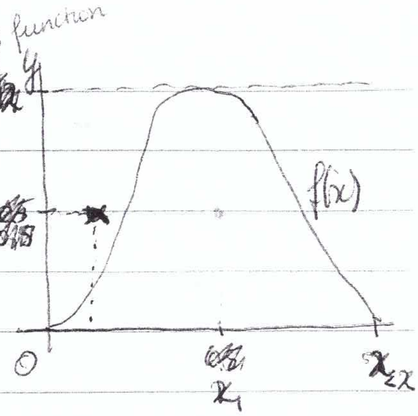
3) The Central Limit Theorem says that if you add up variables (squared) of any distribution (the variables have to come from the same distribution) and the number of variables is very big, the new distribution you get is a Gaussian distribution.
Because this adding happens all the time, almost everything in Nature is taken from a Gaussian distribution.

$$\lim_{N\to\infty} \sum_{i=1}^{N} x_i^2 \Rightarrow \text{Gaussian distr.}$$

true, but also

$$\lim_{N\to\infty} \sum_{i=1}^{N} x_i \Rightarrow \text{Gauss distrib}$$

4) a) You generate random numbers between 0 and 1. These are your y-values. ~~Do~~ Do the same for x-values and ~~multiply by the highest possible f(x)~~ map those to the desired x-range ~~maximum f(x) value~~ to obtain the whole area.



Now use your distribution function: accept any (x_i, y_i) for which $y_i \leq f(x_i)$ and ignore the other points.

× in the graph is rejected, • is accepted.

In this way you can get lots of points under your graph (only) and you can randomly select one of those and use the x-value as your random number.

Also (if your maximum f(x) value is a lot smaller than 1) you can ~~divide the area~~ multiply the random y-values with the max-f(x) value to get the best area to work with.

b) $P(x) = \begin{cases} A(1+ax^2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

Inversion method

First A : $\int_{-1}^{1} A(1+ax^2)\,dx = 1 \Rightarrow A\left[x + \frac{ax^3}{3}\right]_{-1}^{1} = A\left[2 + \frac{a}{3} + \frac{a}{3}\right] = 1$

$2A\left[1+\frac{a}{3}\right] = 1$

$A = \frac{1}{2\left[1+\frac{a}{3}\right]}$

r is unif. distrib between 0 and 1

$p(r) = \begin{cases} 1 & 0 \leq r \leq 1 \\ 0 & \text{otherwise} \end{cases}$; $P(x) = \begin{cases} A(1+ax^2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$\int_{-\infty}^{r} p(r')\,dr' = \int_{-\infty}^{x} P(x')\,dx' \Rightarrow \int_0^r 1\,dr' = \int_{-1}^{x} A(1+ax'^2)\,dx' \Rightarrow$

$\Rightarrow r = \int_{-1}^{x} A(1+ax'^2)\,dx' = A\left(x' + a\frac{x'^3}{3}\right)\Big|_{-1}^{x} = A\left(x + \frac{ax^3}{3} + 1 + \frac{a}{3}\right)$

$\Rightarrow r = A\left(x + \frac{ax^3}{3} + 1 + \frac{a}{3}\right)$ and solve $x = x(r)$

5) Propagate errors using the definition of partial derivatives $f(x,y,z)$

errors: $\Delta x, \Delta y, \Delta z$

$$\sigma_f^2 = \left(\frac{f(x+\Delta x,y,z)-f(x,y,z)}{\Delta x}\right)^2 \Delta x^2 + \left(\frac{f(x,y+\Delta y,z)-f(x,y,z)}{\Delta y}\right)^2 \Delta y^2 + \left(\frac{f(x,y,\Delta z+z)-f(x,y,z)}{\Delta z}\right)^2 \Delta z^2$$

✓ ✓

6) $p(\text{accident}) = 10^{-2}$

probability not having an accident : $1-p$

number of years $N = 30$ probability not accident N years : $(1-p)^N$

I assume involved in at least one accident (≥ 1)

That is: the probability of not being in an accident substracted from 1.

$P(\text{an accident}) = 1-(1-p)^N$

$= 1-(1-10^{-2})^{30} = 0{,}26$ ✓ $P(X>0) = 1-P(X\leq 0) = 1-P(X=0)$

50% chance : $P = 50$

$1-P = (1-p)^N$

$\log(1-P) = N\log(1-p)$

$$N = \frac{\log(1-P)}{\log(1-p)} = \frac{\ln(1-P)}{\ln(1-p)} = \frac{-\ln 2}{\ln(99/100)} = 68{,}97\ yr$$ ✓

That means that to stay under 50%, you can drive for 68 yr.

7) N random variables x_i

$\sum_{i=1}^{N} x_i^2$ This is exactly the χ^2-distribution ✓

When $N \to \infty$ the Central Limit Theorem says that this goes to Gaussian again. ✓